5-27-2003

Dynamics of the Planning Solution in the Textbook Model of Labor Market Search and Matching

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ECONOMICS BULLETIN

Dynamics of the planning solution in the discrete−time textbook model of labor market search and matching

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Abstract

This paper takes a discrete−time adaptation of the continuous−time matching economy described in Pissarides (1990, 2000), and computes the solution to the dynamic planning problem. The solution is shown to be completely characterized by a first−order, non−linear map. We show that the map admits a unique stationary solution which is dynamically unstable. Oscillatory solutions are possible but there is no possibility of periodic solutions. The planner picks the initial condition that places the economy directly on the steady state. Our results are in sharp contrast to received wisdom on out−of−steady−state dynamics in the continuous−time decentralized version of the Pissarides model where adjustment to the steady state is non−instantaneous, and overshooting of vacancies is possible.

We thank an anonymous referee for extremely useful remarks and to Rob Reed for many discussions.
Citation: Bhattacharya, Joydeep and Helle Bunzel, (2003) "Dynamics of the planning solution in the discrete−time textbook model of labor market search and matching." *Economics Bulletin,* Vol. 5, No. 19 pp. 1−10
Submitted: October 10, 2003. **Accepted:** December 23, 2003.
URL: http://www.economicsbulletin.com/2003/volume5/EB−03E00007A.pdf

1 Introduction

This paper takes as a starting point Ljungqvist and Sargent's (2000; Chapter 19) discrete-time adaptation of the continuous-time matching economy described in Pissarides (1990, 2000) and computes the solution to the dynamic planning problem. To the best of our knowledge, we are the first to investigate the dynamic properties of the planning solution in the standard textbook model of labor market search. The solution is shown to be completely characterized by a first-order, non-linear scalar difference equation. The main contribution of this paper is to show that a) there is a unique stationary solution, b) it is locally unstable, and c) oscillatory solutions are possible but there is no possibility of periodic solutions. The implication is strong and clear: in an economy characterized by search and matching frictions, an omniscient social planner has no option but to "jump" to the stationary solution straightaway. These results also suggest that merely delegating the job of coordinating labor market search activity to a planner immediately renders an economy immune to any kind of indeterminacies or endogenous fluctuations. [1]

Several papers in the literature have investigated the possibility of non-stationary solutions in search models of the labor market. The seminal papers in this area are Drazen (1988) and Diamond and Fudenberg (1989); both build on Diamond (1982) and prove the existence of stable limit cycles in a model where there are frictions in coordinating trade, and the matching technology is subject to increasing returns. More recently, Mortensen (1999) revisits the standard textbook model of search and matching in the labor market as described in Pissarides (1990) but introduces an increasing returns to scale production technology to generate multiple long-run unemployment equilibria and stable limit cycles.[2] Shimer and Smith (2001) explore optimal matching policies in constant returns to scale search economies with *heterogenous* agents and find the possibility of non-stationarity.

The current endeavour is different from the previous literature in three important ways. First, the focus here (as in Shimer and Smith, 2001) is on the planning solution as opposed to the "decentralized" solution (the focus of the other aforementioned papers). In particular, there is no free entry condition for firm entry to contend with, nor is there a wage-rent sharing equation derived from protocols of Nash bargaining. To the best of our knowledge, we are the first to investigate the dynamic properties of the planning solution in the standard Pissarides (2000) model. Recall that the dynamics of the decentralized solution (as explored for the continuous-time case in Chapter 1.7 of Pissarides, 2000) is characterized by a system of two differential equations, and that the unique steady state is a saddle point. There, the perfect foresight path in the neighborhood of the equilibrium is unique: a unique set of initial conditions place the economy on the saddle path and non-instantaneous adjustment to equilibrium takes place from there. In fact, overshooting of vacancies is even possible. In contrast, we find that the dynamics of the planning solution is characterized by a scalar non-linear difference equation and the unique steady state is locally unstable. The planner chooses the initial condition to place the economy directly on the steady state; adjustment is instantaneous and no overshooting is possible. Thus there is one and only one planning solution. This last result is of great importance since the famed Hosios condition, a comparison of the social optimum to the market outcome, always done at the steady state which we have now shown is the *only* possible equilibrium.

The second departure from the previous literature is that neither the production nor the matching technologies in our model exhibit any increasing returns. Finally, unlike the continuous-time framework

[1] This result is of independent interest because planning solutions, even in models without any externalities, are not always immune to endogenous fluctuations. See the characterization of chaotic planning solutions to the Ramsey optimal growth model as enunciated in Boldrin and Montruchhio (1986), and more recently in Mitra, Majumdar, and Nishimura (2000).

[2] Mortensen (1999) assumes that match productivity is an increasing function of the aggregate number of matches. This generates the needed increasing returns in production.

used by Mortensen (1999) and Shimer and Smith (2001), we use the discrete-time adaptation.[3] This is important when contrasted again with the decentralized solution to the continuous-time matching economy. There firms create or destroy vacancies instantaneously so as to ensure that the value of a new vacancy is always zero. In our setup, however, vacancies created this period with a cost affect employment *next* period and this introduces a time delay that is not present in the continuous time environment.

The plan for the rest of the paper is as follows. Section 2 outlines the Pissarides (2000) model of search and matching in labor markets. Section 3 contains a detailed analysis of the main difference equation alluded to earlier. Proofs of some central results are to be found in the appendices.

2 The Model

The model (and the notation) is based entirely on Ljungqvist and Sargent's (2000) discrete-time adaptation of the continuous-time matching economy described in Pissarides (1990, 2000). Let $t = 0, 1, 2, 3, ...$ index time. There are two types of agents: workers and firms. There is a continuum of identical workers of measure 1. These workers are all infinitely-lived, they discount the future at the rate β, and are risk-neutral. Workers potentially get matched with a firm; the result of such a match is output y.[4] Each firm may employ at most one worker. A firm incurs a vacancy cost of c in each period when looking for a worker. A match between a worker and a firm gets dissolved with an exogenously-specified probability s. An unmatched worker is an unemployed worker; such a worker enjoys the current utility from leisure of amount z.

Matches are brought together by a standard matching technology connecting only unemployed job seekers with open vacancies. The number of successful matches in a period is given by $\mathcal{M}(u_t, v_t)$ where u_t is the total measure of unemployed workers looking for jobs, and v_t is the number of vacancies or firms looking for employees. The matching function is increasing in both arguments, concave, and homogenous of degree one. For all that we present below, we will assume a standard constant returns to scale formulation,

$$\mathcal{M}(u_t, v_t) = A u_t^{\alpha} v_t^{1-\alpha} \qquad A > 0, \quad \alpha \in (0,1) \tag{1}$$

where A is a scale parameter. The parameter α is the elasticity of the matching function with respect to the measure of unemployed workers. Let $\theta_t \equiv v_t/u_t$ indicate the measure of labor market tightness, or the ratio of vacancies to unemployed workers. Then define $q(\theta_t) \equiv \mathcal{M}(u_t, v_t)/v_t$ denote the probability with which a vacancy gets filled at date t. It follows that $q(\theta) = A\theta^{-\alpha} < 1$ must hold. Similarly, the probability that an unemployed worker gets matched at date t is given by $\theta q(\theta) = A\theta^{1-\alpha} < 1$. Finally, define n_{t+1} as the total number of employed workers at the start of $t+1$. Then, it follows that

$$n_{t+1} = (1-s)n_t + \mathcal{M}(u_t, v_t) = (1-s)n_t + q(\theta_t)\, v_t. \tag{2}$$

The number of people with jobs at the beginning of $t+1$ include the number of undissolved matches which were formed at the start of t that survived onto the start of $t+1$ [given by $(1-s)n_t$] in addition to the new matches formed at t. The following lemma [for proof see Appendix A] will be useful below.

Lemma 1 *Unless $A < 1$ obtains, either $q(\theta) > 1$ or $\theta q(\theta) > 1$ for all θ will be true.*

[3] The discrete-time version of the Pissarides (1990) search-and-matching story has also been employed by Merz (1995), Andolfatto (1996), Shi and Quan (1999), Cooley and Quadrini (1999), Cole and Rogerson (1999), Yuan and Li (2000), and Yashiv (2000), among others.

[4] In the decentralized equilibria, the match surplus is divided between the worker and the firm according to some bargaining protocol. Below we assume that the planner cares only about the match output y, and not the division of the match surplus.

Lemma 1 demonstrates that for the employment probability to be well-defined, A must lie between 0 and 1. This restriction on the scale parameter A is an artifact of the discrete-time nature of the model economy.

A planner's problem could then be outlined as follows. Assume that the planner chooses an allocation that maximizes the discounted value of output and leisure net of vacancy costs. The principal tensions are as follows. An extra vacancy adds a cost, makes it easier for unemployed workers to find jobs, but makes it harder for firms to find workers. Employed workers "lose" leisure utility. More output is produced if the extra vacancy creates more matches. The planner takes all this into account when choosing the number of vacancies. Following Ljungqvist and Sargent (2000; p. 578), the planner chooses v_t and next period's employment level, n_{t+1} by solving (P) where (P) is defined by

$$\max_{\{v_t,n_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[yn_t + z\left(1-n_t\right) - cv_t\right] \tag{P}$$

subject to (2), given a n_0. The Lagrangian can be written as

$$\pounds = \sum_{t=0}^{\infty} \left\{ \beta^t \left[yn_t + z\left(1-n_t\right) - cv_t\right] + \lambda_t \left[(1-s)n_t + q\left(\frac{v_t}{1-n_t}\right) \cdot v_t - n_{t+1}\right]\right\}$$

where λ_t is the Lagrange multiplier on (2). Then, the first-order conditions with respect to v_t and n_{t+1} for an interior solution are given by

$$-\beta^t c + \lambda_t \left[q'\left(\theta_t\right)\theta_t + q\left(\theta_t\right)\right] = 0 \tag{3}$$

and

$$-\lambda_t + \beta^{t+1}\left(y-z\right) + \lambda_{t+1}\left[(1-s) + q'\left(\theta_{t+1}\right)\theta_{t+1}^2\right] = 0 \tag{4}$$

In Appendix B, we show that the second order conditions hold. In addition, we verify that (3) and (4) reduce to the following first-order difference equation in θ :

$$a\theta_{t+1}^{\alpha} - b\theta_{t+1} = \theta_t^{\alpha} - d \tag{5}$$

where

$$a \equiv \beta\left(1-s\right) \in (0,1) \tag{6}$$

$$b \equiv A\alpha\beta \in (0,1) \tag{7}$$

using Lemma 1, and

$$d \equiv \frac{A(1-\alpha)\beta\left(y-z\right)}{c} > 0. \tag{8}$$

Equation (5) is the law of motion for the index of labor market tightness in the economy under the planner's solution. Such a sequence for θ must additionally satisfy $q(\theta) \in (0,1)$, and $n < 1$. As is clear from (5), it is clearly convenient to study the backward dynamics as it is derived explicitly from the first order conditions more easily than the more standard forward dynamics; in fact, θ_t is easily described as a function of θ_{t+1} while the relationship of θ_{t+1} to θ_t is typically a correspondence.[5]

Before proceeding further, it is instructive to contrast the "final product" i.e., eq. (5) with something similar that can be obtained in the *decentralized* continuous-time matching economy of Pissarides (2000).

[5] Analysis of such backward difference equations are common. See for example, Grandmont (1985) and Michener and Ravikumar (1998) for analysis of perfect foresight backward dynamics in the overlapping generations model and a representative agent cash-in-advance model respectively.

As described in Chapter 1 of Pissarides (2000), such an economy may be ultimately reduced to a system of *two differential equations* in the (u, θ) space. There is a unique stationary solution and it is a saddle point. Under perfect foresight, the initial conditions on unemployment and tightness must place the economy on the saddle path from where *non*instantaneous adjustment to the steady state follows. As we show below, eq. (5) in contrast is a one-variable difference equation, with a unique stationary solution; the planner must pick a initial condition to place the economy *instantaneously* on this steady state.

3 Stationary and non-stationary solutions

Given values of n_0 and θ_0 (or v_0), eq. (5) completely characterizes the unique trajectory of θ and all other endogenous variables.[6] In other words, the backwards dynamics of this model can be characterized by the continuous four-parameter family of maps $g : [0, \theta_{\max}] \to [0, g_{\max}]$, where

$$g(\theta) = (a\theta^{\alpha} - b\theta + d)^{\frac{1}{\alpha}}, \qquad (\alpha, a, b, d) \in (0,1) \times (0,1) \times (0,1) \times (0, \infty), \tag{9}$$

and $\theta_{\max} = \left(\frac{\alpha a}{b}\right)^{\frac{1}{1-\alpha}}$ and $g_{\max}$ is implicitly defined as the lowest positive root of the following equation:

$$a g_{\max}^{\alpha} - b g_{\max} + d = 0.$$

The first derivative of the map can be calculated as

$$g'(\theta) = (a\theta^{\alpha} - b\theta + d)^{\frac{1-\alpha}{\alpha}} \left(a\theta^{\alpha-1} - \frac{b}{\alpha} \right), \quad \theta \in [0, \theta_{\max}],$$

which implies that g is unimodal with a unique maximum at $\theta_{\max} \equiv \left(\frac{b}{a\alpha}\right)^{\frac{1}{\alpha-1}}$.

Lemma 2 *The map $g(\theta)$ has a unique stationary solution (θ_{ss}) implicitly defined by*

$$a\theta_{ss}^{\alpha} - b\theta_{ss} = \theta_{ss}^{\alpha} - d.$$

Note that it is not possible to obtain a closed form expression for θ_{ss}. Below, we prove that θ_{ss} is locally unstable.

Proposition 1 *θ_{ss} is asymptotically stable in the non-standard backward dynamics implying it is unstable in the usual forward dynamics.*

The implication is clear and powerful. The only stationary solution to the planner's problem is dynamically unstable. The planner must pick the initial conditions in such a way as to place the economy directly and *immediately* on the steady state. Hence, there is only one solution to the planner's problem and it is the stationary solution.

Finally, we prove that the planning solution may exhibit oscillatory behavior but never any cycles.

Proposition 2

$$-1 < g'(\theta_{ss}) < 1$$

It follows from Proposition 2 that since $g'(\theta_{ss}) < 0$ is possible, the planning solution may exhibit oscillatory behavior; however, since the oscillations are damped in the backward dynamics, they are necessarily *undamped* in the usual forward dynamics. Furthermore, since $g'(\theta_{ss})$ can never equal -1 or go below -1, possibility of two or higher period cycles are ruled out.

[6] Ljungqvist and Sargent (2000; p. 578) assume that the planner just knows n_0. However, only if θ_0 is additionally known can we compute the entire unique solution sequence $\{\theta_t\}_{t=1}^{\infty}$. From there, using (2), it is then possible to compute the optimal sequences $\{v_t\}_{t=0}^{\infty}$, $\{u_t\}_{t=1}^{\infty}$, and $\{n_t\}_{t=1}^{\infty}$. To compute the steady state θ_{ss}, however, information on θ_0 is not needed.

4 Concluding remarks

This paper studies the planning solution to the standard Pissarides (1990, 2000) “textbook model” of search and matching in labor markets. We show that the planning solution is completely characterized by a first-order, non-linear scalar difference equation. There is a unique stationary solution. In fact, it is the only possible solution. The steady state is dynamically unapproachable from a arbitrary set of initial conditions. The planner sets the initial condition to place the economy directly on the steady state. Additionally, the planning solution may exhibit oscillatory behavior but never any cycles. These results are contrasted with those obtained for the decentralized version of the Pissarides (2000) model.

Appendix

A Proof of Lemma 1

Re-writing the conditions that $q(\theta) < 1$ and $\theta q(\theta) < 1$, we get

$$q(\theta) = A\theta^{-\alpha} < 1 \Leftrightarrow \theta > \left(\frac{1}{A}\right)^{-\frac{1}{\alpha}},$$

and

$$\theta q(\theta) = A\theta^{1-\alpha} < 1 \Leftrightarrow \theta < \left(\frac{1}{A}\right)^{\frac{1}{1-\alpha}}$$

That is, θ must lie in the interval $\left(\left(\frac{1}{A}\right)^{-\frac{1}{\alpha}}, \left(\frac{1}{A}\right)^{\frac{1}{1-\alpha}}\right)$. For this interval to be non-empty, we require

$$\begin{aligned}
\left(\tfrac{1}{A}\right)^{-\frac{1}{\alpha}} &< \left(\frac{1}{A}\right)^{\frac{1}{1-\alpha}} \Leftrightarrow 1 < \left(\frac{1}{A}\right)^{\frac{1}{1-\alpha}+\frac{1}{\alpha}} \Leftrightarrow \\
1 &< \left(\frac{1}{A}\right) \Leftrightarrow A < 1.
\end{aligned}$$

If A is greater than one, it must therefore be the case that either $q(\theta)$ or $\theta q(\theta)$ is greater than 1.■

B Second Order Conditions for (P)

It is easy to verify that the second derivatives of the Lagrangian are

$$\begin{aligned}
\frac{\partial^2 \pounds}{\partial v_t^2} &= \frac{\lambda_t}{1-n_t}\left[2q'(\theta_t) + q''(\theta_t)\cdot\theta_t\right] \\
\frac{\partial^2 \pounds}{\partial n_{t+1}^2} &= \frac{\lambda_{t+1}}{1-n_{t+1}}\left[2q'(\theta_{t+1})\cdot\theta_{t+1}^2 + q''(\theta_{t+1})\cdot\theta_{t+1}^3\right] \\
\frac{\partial^2 \pounds}{\partial v_t \partial n_{t+1}} &= \frac{\partial^2 \pounds}{\partial n_{t+1}\partial v_t} = 0
\end{aligned}$$

Therefore the sufficient conditions for a local maximum to obtain are that $\frac{\partial^2 \pounds}{\partial v_t^2} < 0$ and $\frac{\partial^2 \pounds}{\partial n_{t+1}^2} < 0$. Using

$$q'(\theta) = -\alpha A\theta^{-\alpha-1}, \text{ and } q''(\theta) = \alpha(\alpha+1)A\theta^{-\alpha-2},$$

we can write

$$\begin{aligned}
\frac{\partial^2 \pounds}{\partial n_{t+1}^2} &= \frac{\lambda_{t+1}\alpha A\theta_{t+1}^{\alpha+1}}{1-n_{t+1}}\left[-2+(\alpha+1)\right] \\
\frac{\partial^2 \pounds}{\partial v_t^2} &= \frac{\lambda_t \alpha A\theta_t^{-\alpha-1}}{1-n_t}\left[-2+(\alpha+1)\right]
\end{aligned}$$

Clearly both of these are negative since $0 < \alpha < 1$.

C Derivation of Equation (5)

From (3), we obtain the following expression for λ_t :

$$\lambda_t = \frac{\beta^t c}{q'(\theta_t)\,\theta_t + q(\theta_t)} \tag{A1}$$

Using (A1) for period t and period $t+1$, we can insert this expression into (4):

$$-\frac{\beta^t c}{q'(\theta_t)\,\theta_t + q(\theta_t)} + \beta^{t+1}(y-z) + \frac{\beta^{t+1} c}{q'(\theta_{t+1})\,\theta_{t+1} + q(\theta_{t+1})}\left[(1-s) + q'(\theta_{t+1})\,\theta_{t+1}^2\right] = 0$$

Inserting the expressions for $q(\theta)$ and $q'(\theta)$ (recall that $q(\theta) = A\theta^{-\alpha}$) and re-arranging, we get:

$$\beta(y-z) + \frac{\beta c}{-\alpha A\theta_{t+1}^{-\alpha} + A\theta_{t+1}^{-\alpha}}\left[(1-s) - \alpha A\theta_{t+1}^{-\alpha+1}\right] = \frac{c}{-\alpha A\theta_t^{-\alpha} + A\theta_t^{-\alpha}} \tag{A2}$$

From here, straightforward manipulation yields

$$(1-s)\,\beta\theta_{t+1}^{\alpha} - \alpha A\beta\theta_{t+1} = \theta_t^{\alpha} - \frac{(1-\alpha)\,A\beta\,(y-z)}{c},$$

which immediately provides the desired expression in (5).■

D Proof of Lemma 2

Re-writing the equation which defines the steady state, we get:

$$(a-1)\,\theta_{ss}^{\alpha} + d = b\theta_{ss} \tag{A3}$$

First note that the function on the left hand side of (A3) takes the value $d > 0$ when $\theta = 0$, while the right hand side of (A3) starts out at 0. Further more the left hand side is strictly decreasing, as $a < 1$, and the right-hand side is strictly increasing as $b > 0$. This implies that if a steady state exists, it will be unique.

Furthermore, since $a < 1$

$$\lim_{\theta\to\infty}\left((a-1)\,\theta^{\alpha} + d\right) = -\infty$$

and since $b > 0$

$$\lim_{\theta\to\infty} b\theta = +\infty,$$

there exists a $\bar{\theta}$ such that

$$(a-1)\,\bar{\theta}^{\alpha} + d < b\bar{\theta}.$$

Since both the left hand side and the right hand side are continuous, a standard fixed point argument will establish that a steady state exists. We have thus established that the map $g(\theta)$ has a unique stationary solution.■

E Proof of Propositions 1 and 2.

Note that if $g'(\theta_{ss}) \in (-1,1)$, the map is stable in the backwards dynamics, and therefore it is unstable in the forward dynamics. Therefore the proof of both propositions will be complete if we can show that $g'(\theta_{ss}) \in (-1,1)$. Using the definition of θ_{ss}, namely that $\theta_{ss} = (a\theta_{ss}^{\alpha} - b\theta_{ss} + d)^{\frac{1}{\alpha}}$, as well as the definition of the parameters, we get the following expression for $g'(\theta_{ss})$:

$$\begin{aligned} g'(\theta_{ss}) &= (a\theta_{ss}^{\alpha} - b\theta_{ss} + d)^{\frac{1-\alpha}{\alpha}} \left(a\theta_{ss}^{\alpha-1} - \frac{b}{\alpha} \right) = \theta_{ss}^{1-\alpha} \left(a\theta_{ss}^{\alpha-1} - \frac{b}{\alpha} \right) \\ &= \left(a - \frac{b}{\alpha}\theta_{ss}^{1-\alpha} \right) = \beta(1-s) - A\beta\theta_{ss}^{1-\alpha} = \beta(1 - s - \theta_{ss} q(\theta_{ss})) \end{aligned}$$

Now, since $0 < \beta < 1$, $0 < s < 1$, and $0 < q(\theta) < 1$, it is immediately clear that $g'(\theta_{ss}) \in (-1,1)$. $\blacksquare$

References

[1] Andalfatto, D, (1996). “Business cycles and labor market search” *American Economic Review*, **86**(1), 112-32.

[2] Boldrin, M., and L. Montruchhio (1986) “On the Indeterminacy of Capital Accumulation Paths” *Journal of Economic Theory*, **40**, 26-39

[3] Cole H.L., Rogerson, R., 1999. “Can the Mortensen-Pissarides matching model match the business-cycle facts?” *International Economic Review*, **40(4)**, 933-59.

[4] Cooley, T.F., Quadrini, V. (1999). “A neoclassical model of the Phillips curve relation” *Journal of Monetary Economics*, **44** (2), 165-193.

[5] Diamond, P.A. 1982. “Aggregate demand management in search equilibrium” *Journal of Political Economy* **90**, 881-94

[6] Diamond, P.A., Fudenberg, D. 1989. “Rational expectations business cycles in search equilibrium” *Journal of Political Economy* **97**, 606-619.

[7] Drazen, A., 1988. “Self-fulfilling optimism in a trade-friction model of the business cycle” *American Economic Review*, **78(2)**, 369-72.

[8] Grandmont, J.-M., 1985. “On endogenous competitive business cycles” *Econometrica* **53**, 995–1045.

[9] Ljungqvist, L. and Sargent, T. (2001) *Recursive Macroeconomic Theory*. MIT Press, Cambridge: Massachusetts

[10] Merz, M. (1995) “Search in the labor market and the real business cycle” *Journal of Monetary Economics*, **36**(2), 269-300.

[11] Mortensen, D. (1999) “Equilibrium unemployment cycles” *International Economic Review*, **40**, 889-914

[12] Michener, R. and Ravikumar, B. (1998) “Chaotic dynamics in a cash-in-advance economy” *Journal of Economic Dynamics and Control*, **22** (7), 1117-1137.

[13] Mitra, T. , M. Majumdar, and K. Nishimura (eds.) (2000). *Optimization and Chaos*, Springer-Verlag.

[14] Pissarides, C.A. 1990. *Equilibrium Unemployment Theory*, Oxford: Blackwell.

[15] —— 2000 *Equilibrium Unemployment Theory*, MIT Press: Cambridge Massachussetts

[16] Shi, S., Quan W., (1999) “Labor market search and the dynamic effects of taxes and subsidies” *Journal of Monetary Economics*, **43** (2), 457-495.

[17] Shimer, R and L. Smith (2001) “Non-stationary search” mimeo Princeton University

[18] Yashiv E. (2000) “The determinants of equilibrium unemployment” *American Economic Review*, **90(5)**, 1297-1322.

[19] Yuan, M., Li, W. (2000) “Dynamic employment and hours effects of government spending shocks” *Journal of Economic Dynamics and Control*, **24** (8), 1233-1263